

## **Title: Geometry – Monkeying Around with Similar and Congruent Figures**

### **Brief Overview:**

In this unit, students will investigate the properties of congruent and similar two– and three–dimensional figures. Students will apply their geometry and measurement skills through hands–on activities connected to area, perimeter, and volume.

Note: It is assumed that various concepts such as scale factor, proportions, and volume have been previously taught. It may be necessary to supplement/modify the lessons in order to meet the needs of students.

### **NCTM Content Standard/National Science Education Standard:**

#### Geometry

- Analyze characteristics and properties of two– and three–dimensional geometric shapes and develop mathematical arguments about geometric relationships.

#### Measurement

- Apply appropriate techniques, tools, and formulas to determine measurements. Understand measurable attributes of objects and the units, systems, and processes of measurement.

### **Grade/Level:**

Grade Seven, Geometry

### **Duration/Length:**

Five 45 – 50 minute lessons

### **Student Outcomes:**

Students will:

- Understand relationships among the angles, side lengths, and perimeters of similar objects.
- Understand relationships among the areas of similar objects.
- Understand relationships among the volumes of similar figures.
- Apply conceptual knowledge of area, perimeter, and volume of similar figures to real life situations.

### **Materials and Resources:**

- Overhead Geoboard

- Calculators
- Envelopes
- Group sets of:
  - Rectangles (Two congruent and two similar)
  - Squares (Two congruent and two similar)
  - Triangles (Two congruent and two similar)
  - One triangle neither congruent nor similar to the above
- Note cards (3 per group)
- Rulers
- Protractors
- Multi-colored sticky dots
- Centimeter Dot Paper
- Geoboard for each student/group and rubber bands
- Centimeter Cubes
- Pictures of local zoos and animals
- Graph paper
- Plain white paper
- Markers, crayons, or colored pencils
- Computers (as assigned by teacher)
- Worksheets
  - Venn Diagram
  - Congruent and Similarity Sorting Activity
  - Plus/Delta and Consensogram
  - Area Exit Ticket
  - Table of Similar Figures
  - Table of Similar Solids
  - Volume: It's a Zoo Out There

### **Development/Procedures:**

#### **Lesson 1**

Pre-assessment – Ask the students the following key questions:  
 “What does it mean if a shape is congruent? Similar?”

Launch – Use an overhead Geoboard to show students rectangles, squares, and triangles. Have students volunteer to name the shape and verbalize what characteristics make the shape a rectangle, square, or triangle. Ask the students how to find the area and perimeter of each. Encourage students to use “math words”, but do not give them the vocabulary. Vocabulary will be developed as the lessons progress.

Teacher Facilitation – Prior to class, set up a class set of envelopes, where each envelope contains four of each of the following shapes: rectangles, squares, and triangles. For each shape, have two congruent and two similar shapes. At the beginning of class, divide the class into small groups and distribute the envelopes.

Also, hand out a Venn Diagram and a copy of the “Congruent and Similarity Sorting Activity” worksheet. Students are to read the directions on the worksheet and begin the assignment. Have students complete a Venn Diagram to reinforce the information they discover about congruent and similar figures. Be sure to make enough copies of the Note Data Cards from the worksheet, and also make index cards available for students to attach to the note card.

Student answers should be very simple for lesson one. As the unit progresses, students will add to their Venn Diagram and answers will become more in-depth as their understanding increases. Students will present their findings and conclusions to the class. During this time, introduce the math vocabulary that you want the students to use in their discussions and written explanations.

Student Application – Students assign each member of their group a job as described in the directions from the above “Congruent and Similarity Sorting Activity”. Students then separate the shapes into categories. Using their note cards, students begin to name their categories and list characteristics that they have in common for that category. Students should be reminded to use “math words” and look at the side lengths as well as the angle measurements. Students will then find two shapes in each category that are similar. Students will measure their side lengths and determine all angle measurements. Students will find the perimeter and area of each shape and begin to make conjectures as to what happens when the side lengths change. Students should begin to see how scale factor is related to perimeter and area. (It is assumed that scale factor has been covered at this point. If students have not been exposed to this concept, teacher may need to facilitate discussion of scale factor). Let the students know that they will be investigating area in the next lesson.

Embedded Assessment – Practical application of concepts will be demonstrated and assessed through the completion of the sorting activity and class discussion to follow. Students will also complete a “Plus/Delta and Consensogram” evaluating today’s lesson. Students may also complete the Area Exit Ticket to reinforce the use of formulas in mathematics. Further assessment will be demonstrated in Lesson Two as students use the Geoboards to practice examples of perimeter and area.

Reteaching/Extension - Based on the results of the Plus/Delta and Consensogram, the teacher should be able to see the confidence levels and problem areas students are having. For students that are having difficulty, the lesson will be reinforced in Lesson Two using Geoboards. Students may also use their graphing calculators to perform necessary calculations and record in their journals the information presented using the Area Formula Application. Student progress may also be measured by the completion of the Quiz in the Area Formula Application found on TI-84 Graphing Calculators.

For students that have an understanding of the lesson, they may begin exploring in Cabri® Junior on their graphing calculators. Students may also apply the concepts of similarity and congruence, and perimeter and area by developing real life situations where these concepts are used.

## Lesson 2

Pre-assessment – During Lesson One, students discovered that when the side lengths of two dimensional figures are increased by the same scale factor, then the resulting similar figure has a perimeter that has also been increased by that same scale factor. Students also discovered that the area of the similar figure increased or decreased as well, but not by the same scale factor.

Have students return to their groups. Allow time for students to review their Venn Diagrams and formulate questions to explore. The speaker of the group will present their findings and questions to the class in a whole group setting. This will allow the teacher to assess the previous days learning.

Launch – Using the previous day's information, students will work in small groups to try to analyze the information collected and make a conjecture as to why the area increased by a different amount. If this conjecture came at the end of day one, then a review discussion will take place to remind student of the activity and to get students who may have been absent caught up.

Teacher Facilitation – The teacher will use the overhead projector to model for students how to use the Geoboard and/or dot paper to determine area. Students will practice using their own Geoboard and/or dot paper.

Student Application – Students will work in groups of two or three to complete the worksheet, "Table of Similar Figures for Rectangles". One student will use the Geoboard to model the original figure and find its area. One student will model the scale

factor figure and find the area. The third student will check the first two students' work and record information in the table. Each student in the group will need to copy the table.

When students have completed this activity, the teacher will reconvene the group for a brief discussion about the tables. It is important that all students have correct information in their table. Students will then continue in their groups to compare the area of the "original" figure to the area of the "similar" figure. Ask students to predict what they think will happen with the area of triangles. The previous three steps are repeated to complete the second worksheet, "Table of Similar Figures for Triangles". Monitor the group and class discussions, offering suggestions if students seem to need a nudge in the right direction. If needed, suggest that they look at the ratio between the area of the original and the area of the similar figure.

Students will use the 'In your own words...' section at the bottom of their "Table of Similar Figures" worksheets to describe their findings. The lesson will conclude with a whole-class discussion on the findings from each group and adding additional information to the Venn Diagram about the area of similar and congruent figures.

Embedded Assessment – Continuous assessment will take place during each phase of the lesson by way of discussion and teacher monitoring.

Reteaching/Extension - The description at the bottom of the tables of similar figures will serve as a gauge of student understanding. If needed this lesson can be reviewed/re-taught with the activities being more teacher-directed.

Students who are ready for an extension could choose a favorite cartoon character and create an enlargement of it by a given scale factor. Students will first grid the original picture and then create a new grid for the given scale factor. Students then copy each box of the grid to the new grid in proportion to create an enlargement of the original picture.

### Lesson 3

Pre-assessment – A review of the findings from Lesson 2 will serve as the pre-assessment for this lesson. Students should have a good understanding that when the length of the sides of a figure increases by a given scale factor, then the area of the resulting similar figure has increased by the square of that scale figure. Have

students return to their groups. Allow time for students to review their Venn Diagrams and formulate questions to explore. The speaker of the group will present their findings and questions to the class in a whole group setting. This will allow the teacher to assess previous days learning.

Launch – Based on the results of the investigations from Lessons 1 and 2, students will be asked to make a conjecture about how the volume of similar solids relates, focusing on rectangular prisms only. Students may need a reminder about the concept of volume and the formula for finding volume.

Teacher Facilitation – Ask students what is different about finding volume than what they did on the previous days. Students should eventually decide that there is another dimension that needs to be used. Ask students what units are given to the answers when finding the volume of three-dimensional figures. Review linear units, square units, and cubic units. Model the concept of volume by filling an empty rectangular solid with inch cubes and centimeter cubes.

Student Application – Students will again work in groups to complete the “Table of Similar Solids” worksheet. They will first trace the length and width of the solid on the centimeter graph paper. Then they will build the solid using the centimeter cubes. When the constructions are complete, and all students in the group have verified that the dimensions are correct, it will be disassembled in order to count the cubes. Students will be asked to compare the volume of the original to the volume of the similar figure. A hint is provided again for them to use the ratio of the volume of the two solids.

Listen in to each group and give appropriate guidance as needed. When the groups have completed the activity, whole class discussion will take place. This will give the students the opportunity to clarify their thoughts before they are asked to write about it.

After discussion, each student will complete the “In your own words,” portion of the table. Once the students have completed this description, the students in the group will pass their paper clockwise to another member. Each member of the group will review the other person’s entire paper for completeness and clarity,

and to ask any questions about it. If needed, the owner of the paper will be given the opportunity to revise anything needed.

Students will return to the Venn Diagram to add additional information about volume of congruent and similar figures.

Embedded Assessment – Assessment takes place throughout the assessment as the teacher rotates from group to group, conducts class discussions, and asks students clarification questions. Additionally, the final activity in groups will provide time for students to practice using the appropriate math terms in descriptions and explanations.

Reteaching/Extension – The teacher will use the tables and written explanations to guide any re-teaching or remediation needed.

If time allows, this lesson could be extended to include examples of the triangular prism. This would be more difficult to model with the centimeter cubes. Students ready for extension could build the rectangular prism and then visualize the “cutting” off of the triangular prism.

If available, use graphing calculators or the computer lab to let students actually see the relationship between similar figures and their corresponding areas. One program with this capability is Cabri® for computers and Cabri® Junior for graphing calculators. The teacher should be comfortable with the program before beginning.

#### Lesson 4

Pre-assessment – Students will review concepts from Lessons 1–3 by revisiting their Venn Diagrams. Have students return to their groups. Allow time for students to review their Venn Diagrams and formulate questions to explore. The speaker of the group will present their findings and questions to the class in a whole group setting. This will allow the teacher to assess the previous days learning.

Launch – Show students pictures of local Zoos in the area. Ask students: “How could we relate what we have learned in the last week to the pictures I have just shown you?” Have students brainstorm ideas of how math can be related to the real world. Discuss different career choices and positions available to people who are gifted in math, such as architects, designers, teachers, chemists, cryptologists, author, the TV series “Numbers”, etc.

Once students have sufficiently covered relating similarity, area, perimeter, and volume to real life examples, begin to introduce the Zoo Project.

Teacher Facilitation – Hand out “Volume: It’s a Zoo Out There” to students. Read through the instructions and directions with students. Take time to clarify any questions students may have. Explain that students may work independently or in pairs to complete this project. Teacher should be available throughout the project to help students, and should actively monitor student progress. Set a timetable up with students.

Student Application – Students will begin to complete the project outlined in “Volume: It’s a Zoo Out There”.

Embedded Assessment – Student mastery of this lesson will be assessed through the completion and accuracy of the culminating activity, their “Zoo Design”.

Reteaching/Extension – As an extension, students may work under the restraints of a budget. Students will be assigned a given amount of money, and must design a budget around the costs of production.

Technology connection – Students may complete the project on the computer using the appropriate technology available at your school.

Science Connection – Students may access databases including information on animal habitats and complete a brief research paper explaining why they assigned a certain amount of area to each animal.

### **Summative Assessment:**

The concepts taught throughout this lesson will be assessed through the culminating activity.

### **Authors:**

April Alcorn  
North East Middle School  
Cecil County

Janet Gregory  
Southern Middle School  
Garrett County